Comparative Study of Interpolation Methods in Development of Local Geoid

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Abstract
The main aims of the research are to estimate the efficiency of different methods of interpolation (Natural Neighbour, Inverse Distance Weighting, Kriging and Topo to Raster) for the normal height determination by comparing with heights anomalies determined by using GPS-leveling and global gravitational model EGM2008. The discrepancies between the calculated and interpolated heights were analyzed and discussed for Fergana valley located within in Republic of Uzbekistan. Comparison of the results with the data of global gravimetric model EGM 2008 showed that the best interpolation surface demonstrated Natural Neighbour and Topo to Raster methods for considered region.

1. Introduction

Geodetic Network of Uzbekistan is fragment of Commonwealth of Independent States Geodetic Network. National height system of Uzbekistan are based on Krasovsky ellipsoid and referenced to the mean sea level with zero-mark of the Kronstadt tide gauge in Pulkovo. The Baltic normal height system was adopted in 1977 and is in use up to nowadays (Belevich and Bekbaev, 2006). With the purpose of improving the national geodetic reference frame an establishment of the new State Geodetic Network (SGN), based mainly on use the GPS measurements, is carried out in Uzbekistan since 2013 (RP-2045, 2013). This project plans to establish and maintain a Continuously Operating Reference Station (CORS) network of 50 Reference Stations across the country (Fazilova and Magdiev, 2016). The obtained horizontal coordinates of the network will be used directly in engineering applications and large scale map production. But the vertical component (ellipsoidal height) must be before transformed into normal heights. There can be considerable difference in values of the normal height and ellipsoidal height in control points across territory of the country due relief futures. Geoids heights must be known with required accuracy for transformation procedure. Unfortunately, there is no systematic data base of normal heights definition on territory the country by modern methods also.

Recently, the new gravity satellite missions provide global solutions that allow modeling the long and medium wavelengths of the Earth's gravitational field. Such models, as the EGM2008 (Pavlis et al., 2012) solution (Earth Gravitational Model released in 2008), represent a major advance in the geodesy and provide geoids heights with respect to WGS- 84 ellipsoid. A common problem faced by most countries is the definition of local geoid by using the global gravitational model.

Due to the lack and irregular of normal heights data for this moment, naturally, there is a problem of interpolation of the available classical data set on whole territory. The main aims of the research are to estimate the efficiency of different methods of interpolation (Natural Neighbor, Inverse Distance Weighting, Kriging and Topo to Raster) for the normal height determination by comparing with heights anomalies determined by using GPS-leveling and global gravitational model EGM2008. The discrepancies between the calculated and interpolated heights were analyzed and discussed.

2. Data and Method

We considered 3 kinds of data for carrying out the analysis:

a) ellipsoidal heights of 9 benchmarks have been obtained from 2010 to 2015 through SGN observation project from GPS measurements in Fergana valley in Uzbekistan (39°-44°N latitude and 65°-75° E longitude) (Ergeshev et al., 2016) (Figure 1).

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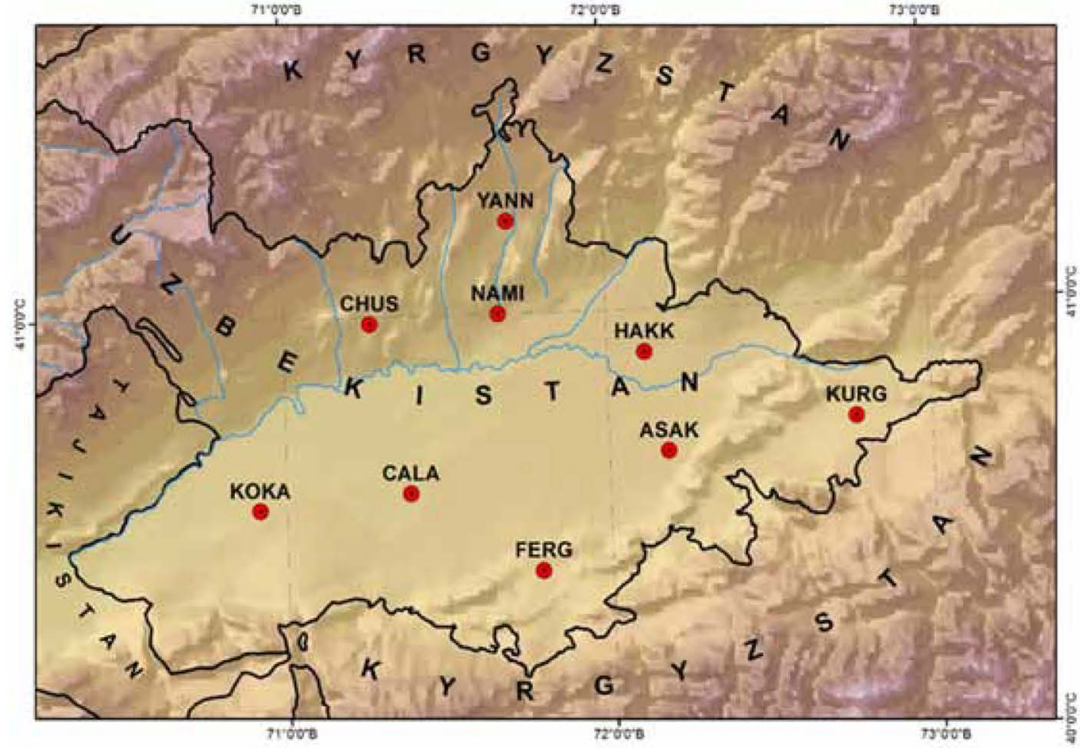

Figure 1: GPS network in Fergana valley

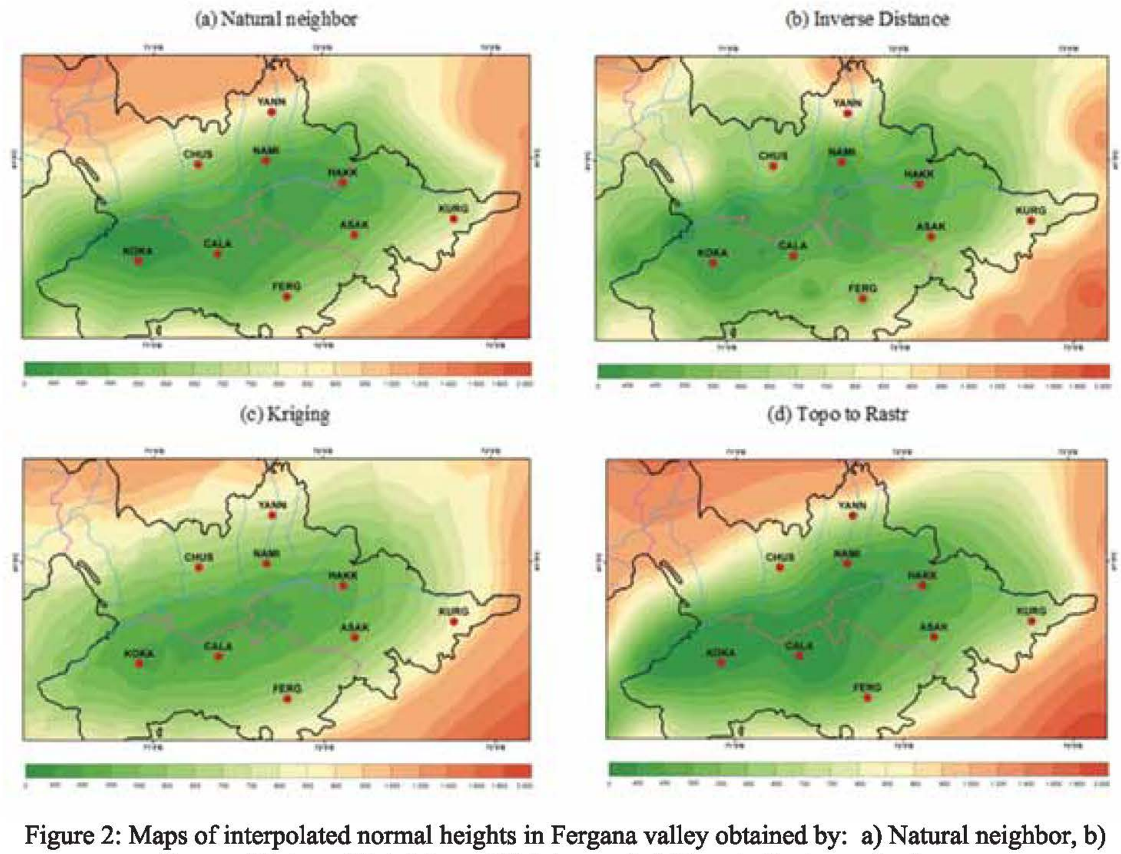

Figure 2: Maps of interpolated normal heights in Fergana valley obtained by: a) Natural neighbor, b) Inverse Distance Weighting, c) Kriging, and d) Topo to Raster methods

b) classic leveling of 1 order in considered part of the Fergana valley during the period from 1911 to 1936 was done by different scientists of Tashkent observatory and former Military topography department such as Zalesky, Pomerantscev (Sheglov, 1989). Results of measurements of normal heights in the area (in total 242 points) available in the database of the International Gravimetric Bureau in Toulouse were used for the analysis (Drewes et al., 2016).

c) geoid undulations for EGM2008 at control points are received using Calculation Service of the International Center of Global Terrestrial Models (ICGEM) hosted by GFZ German Research Centre for Geosciences at Potsdam (Barthelmes and Köhler, 2016). This gravitational model is complete to spherical harmonic degree and order 2159. The model was computed from a global 5 arc-value minute grid of gravity anomalies from land and satellite-based sources. ICGEM service allows to calculate geoid heights above WGS-84 ellipsoid for EGM2008 model with 1°x1°grid. Geoid height at a point P defined by its geocentric distance (r), geocentric co-latitude (φ), defined as (90°-latitude), and longitude (λ) is given by (Heiskanen and Moritz, 1967):

$$\zeta(B,L,H) = \frac{GM}{\gamma(B,L,h)r}\sum_{n=2}^{N}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}\bar{P}_{nm}(sin\varphi) * (\bar{C}_{nm}cos\lambda + \bar{S}_{nm}sinm\lambda),$$

Equation 1

where B, L, H –ellipsoidal WGS-84 coordinates of a point, h- normal height of a point, fM – geocentric gravitational constant, a- equatorial radius of WGS-84 reference ellipsoid, γ-mean normal gravity on the surface of the reference ellipsoid, $P_{nm}(sin\varphi)$ - fully-normalized associated Legendre functions, C_{nm},S_{nm}- fully-normalized harmonic coefficients of the disturbing potential. The infinitive series is usually truncated at the maximum degree (N) of the expansion n (n, m – degree, order of spherical harmonic respectively).

The analysis of normal height interpolation accuracy is based on estimating difference at identified control point P:

$$\Delta\zeta(P) = \zeta_H(P) - \zeta_{EGM2008}(P)$$

Equation 2

Where $\zeta_H(P) = H^{GPS}(P) - h^{classical}(P)$ - geoid height (undulation) (Hofmann-Wellenhof et al., 2001), $H^{GPS}(P)$ - ellipsoidal height of GPS benchmarks, $h^{classical}(P)$ - interpolated by using different methods normal height from BGI data base,$\zeta_{EGM2008}(P)$ – EGM2008 geoid height at point P calculated by using Equation 1.

The ArcGIS 9 Spatial Analyst tool has been used for creation of a required normal height surface model (Childs, 2004). The tool provides a set of deterministic and geostatistical methods for performance of spatial interpolation, allowing the user to receive results for territories with the lacking or missed data. Deterministic interpolation techniques create surfaces based on measured points or mathematical formulas. Methods such as Inverse Distance Weight (IDW) are based on the extent of similarity of cells. Geostatistical interpolation techniques such as Kriging are based on statistics and are used for more advanced prediction surface modeling that also includes some measure of the certainty or accuracy of predictions (Childs, 2004). Advantages and disadvantage of all interpolation methods were discussed in (Dumitru et al., 2013). Different studies have been performed also for analysis of accuracy interpolation methods (Soycan and Soycan, 2003). Different interpolation methods were used during last years for determination geoid quantities also. In (Erol and Celik, 2004), for example, demonstrated that Kriging method fits data better than IDW interpolation for local geoid modeling using GPS/leveling data in Turkey. For geoid – quasigeoid separation in Iran it was proved that "triangulation with linear interpolation", "nearest neighbor" and "radial basis function" methods can be applied for different regions of country depending from topography (Mehramuz et al., 2012).

In this research, normal height values are estimated in the internal points of the studied region. We considered, taking into account abovementioned experiences, Natural Neighbor (NN), Inverse Distance Weighting (IDW), Kriging (KR) and Topo to Raster (TR) methods for interpolating of normal height surface. In the first step, normal height values were interpolated on Fergana valley field by using different methods, which description given below and, respectively, values for control points were estimated. Then the evaluation of the interpolated normal heights was carried out by comparing with values received from differences between geoid heights derived from EGM2008 and values of GNSS leveling benchmarks above WGS-84 ellipsoid.

3. Results

In order to generate continuous areas for research, classical normal heights from BGI data base was interpolated by using different methods. Results of interpolation of normal height ($h^{classical}$) by NN, IWD, KR and TR methods are presented on Figure 2. The differences between the four chosen methods of interpolation are clearly visible in the two-dimensional graphic representation. Figure 2 shows that results for valley part (ASAK, GALA, KOKA, NAMI, HAKK) of investigated region is almost the same, while the results of interpolation for mountainous part (KURG, CHUS, YANN, FERG) greatly depends on the model developed by interpolation. Irregularly distributed leveling data, especially in mountainous regions, is certainly influencing on the result of interpolation also.

Table 1 shows height values GPS benchmarks of Fergana valley network points and EGM2008 geoid undulation estimated by the ICGEM.

Table 2 shows calculated the differences between normal heights by using Equation 2), applied to four different methods of interpolation. The results demonstrate an error level of 17 meters to 47 meters, while the minimum deviation varies from -23 to -94 meters, and the maximum difference ranges from 9 to 47 meters. It was concluded that Nearest Neighbour method gives more accurate result for territory of Fergana valley.

Table 1: Ellipsoidal height and geoid undulation estimated by using EGM2008 model for points in Fergana valley

Station	**Latitude B,°**	**Longitude L,°**	H^{GPS}**, m**	$\zeta_{EGM2008}$**, m**
ASAK	40,6655	72,1860	437,589	-47,73590012
GALA	40,5884	71,3884	347,265	-48,57845375
KOKA	40,5437	70,9197	366,248	-47,40629863
KURG	40,7288	72,7711	678,379	-44,70985098
NAMI	40,9937	71,6625	383,22	-47,50214008
HAKK	40,9062	72,1140	397,971	-48,01803154
CHUS	40,9892	71,2622	595,747	-46,14539992
YANN	41,2167	71,7014	737,545	-45,05621435
FERG	40,3945	71,7684	528,208	-46,33632449

Table 2: Differences (m) between height values obtained by using the interpolation methods and those obtained from the differences between ellipsoidal height and geoid undulation estimated by using EGM2008 model for whole investigated region

Station	$\Delta\zeta$**, m**			
	NN	**KR**	**IDW**	**TP**
ASAK	-35,6451	-61,3351	-36,2731	-23,6421
GALA	-15,7075	-63,9285	-68,2415	24,3755
KOKA	17,5253	-39,0817	-0,1957	43,2063
KURG	1,0708	-94,4412	-52,4921	42,6509
NAMI	3,0091	-93,4589	8,9881	11,8561
HAKK	3,8710	-33,7870	4,5140	6,1010
CHUS	23,2184	34,2544	9,1104	22,9424
YANN	2,0052	31,4292	-41,5348	47,8752
FERG	6,0403	-68,2697	-0,6647	8,8483
Standard deviation,m	**17,4141**	**47,7139**	**29,9273**	**22,7567**
Min, m	**-35,6451**	**-94,4412**	**-68,2415**	**-23,6421**
Max, m	**23,2184**	**34,2544**	**9,1104**	**47,8752**

It should be noted that Topo to Raster (RMS=22 m) combines the computational efficiency of Inversely Weighted Distances, Kriging and allows take into account changes in the Earth's surface (such as gorges, ridges, steep cliffs). Preliminary results showed that NN and TR interpolation methods gave reasonable results for this local area and they are applicable for modeling the geoid of this area as surface for practical geodetic applications.

4. Conclusions

The present study has evaluated normal height interpolation methods (Natural Neighbor, Inverse Distance Weighting, Kriging and Topo to Raster) over Fergana valley part located in Uzbekistan as an example for development of local geoid in future. Normal heights evaluation values for EGM2008 sourced from two different online free data services: Calculation Service of the International Center of Global Terrestrial Models hosted by GFZ German Research Centre for Geosciences at Potsdam and International Gravimetric Bureau in Toulouse (France). The obtained data were compared with new GPS measurements data in the country.

The results of this research show that the output results depend on the density data, interpolation methods, terrain features. It should be noted that it is necessary to develop an optimal SGN design especially for the mountain regions of the country. The comparisons involving the astrogeodetic data provide evidence that EGM2008 can be used for preliminary geoid construction works over Fergana valley. A significant step in development of the national geoid became the document adopted in December 2017 in Uzbekistan (Decree №1022, 2017). As a general conclusion, it can be concluded it is necessary to be supported also with precise gravimetric, GPS measurements in the country.

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